Answer all of the following questions. Calculators, Pagers and mobile telephones are NOT allowed.

1. Let $f(x) = x^2 - 4x + 5$, $x \geq 2$.

   Show that f^{-1} exists and state its domain and range and compute $f^{-1}(x)$. (4 points)

2. (a) Use logarithmic differentiation to find $\dfrac{dy}{dx}$ if

$$y = \frac{(5x^4 - 3x)^3 \sec^2 x}{e^{(x^2+1)} \sqrt[3]{1 + x^2}}.$$ (4 points)

   (b) Find $\dfrac{dy}{dx}$ where

$$\sin^{-1}(xy) - \tan^{-1}\left(\frac{x}{y}\right) = 0.$$ (4 points)

3. (a) Show that

$$\sin^{-1}(\tanh x) = \tan^{-1}(\sinh x) \qquad \text{for all } x.$$ (4 points)

   (b) Find

$$\lim_{x \to \infty} \left(\frac{x}{x - 3}\right)^x, \text{if it exists.}$$ (4 points)

4. Evaluate the following integrals (4 points each)

   (a) $\displaystyle \int \frac{1}{x\sqrt{1 - (\ln x)^2}}\, dx$

   (b) $\displaystyle \int \sqrt{x}\, \ln x\, dx$

   (c) $\displaystyle \int \sqrt{\sin x}\, \cos^3 x\, dx$

   (d) $\displaystyle \int \frac{x - 3x^2}{\sqrt{x^2 - 16}}\, dx$

   (e) $\displaystyle \int \frac{x^6 - x^3 - 54}{x^4 - 9x^2}\, dx$

Good Luck